Sample Questions

Don't use calculators.

1. (a) Write down formulae for the quotient q and remainder r produced by the division algorithm when an integer a is divided by a positive integer b (so that $a = qb + r$ where $0 \leq r < b$).

   (b) Apply the division algorithm to the integers $a = 73$ and $b = 11$ to produce the quotient q and remainder r when a is divided by b.

   (c) Repeat (b) with the values of a and b changed to 60 and 9 respectively.

2. (a) Use the Euclidean algorithm to find the gcd (greatest common divisor) d of $a = 73$ and $b = 11$.

   (b) Express d as a linear combination $d = xa + by$ of a and b, where x and y are integers.

3. (a) Use the Euclidean algorithm to find the gcd (greatest common divisor) d of $a = 120$ and $b = 35$.

   (b) Express d as a linear combination $d = xa + by$ of a and b, where x and y are integers.

4. Consider the prime number algorithm described in Lecture 17.

   (a) Suppose that the table has been completed to the stage where the first eight prime numbers have been produced, with the last one being $p_8 = 19$. Continue the table for $k = 9$ to show how the algorithm produces the next prime number p_9.

   (b) Continue the table to show how the algorithm produces p_{10}.

5. (a) Prove by induction that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

   for every positive integer n.

   (b) Prove by induction that

$$1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots + \left(-\frac{1}{3}\right)^n = \frac{3}{4}\left[1 - \left(-\frac{1}{3}\right)^{n+1}\right]$$

   for every non-negative integer n.

6. (a) In the Tower of Hanoi puzzle, what is the least number of moves (shifting one disc at at time) that is required when $n = 2$? (That is, we want to move 2 discs from the first pole to the third pole.)

   (b) Repeat (a) but now with $n = 3$.